1.7 Solving Absolute Value Inequalities

Review of the Steps to Solve a Compound Inequality:

- Example: $2x+3>2$ and $5x<10$
- This is a conjunction because the two inequality statements are joined by the word "**and**".
- You must solve each part of the inequality.
- The graph of the solution of the conjunction is the intersection of the two inequalities. Both conditions of the inequalities must be met.
  - In other words, the solution is wherever the two inequalities overlap.
  - If the solution does not overlap, there is ***no solution.***

Review of the Steps to Solve a Compound Inequality:

- Example: $3x \leq 15$ or $-2x+1 \geq 0$
- This is a disjunction because the two inequality statements are joined by the word "**or**".
- You must solve each part of the inequality.
- The graph of the solution of the disjunction is the union of the two inequalities. Only one condition of the inequality must be met.
  - In other words, the solution will include each of the graphed lines. The graphs can go in opposite directions or towards each other, thus overlapping.
  - If the inequalities do overlap, the solution is ***all reals.***

"*and*" Statements can be Written in Two Different Ways

- 1. $\mathbf{8 < m + 6 < 14}$

- 2. $\mathbf{8 < m+6}$ **and** $\mathbf{m+6 < 14}$

These inequalities can be solved using two methods.

Method One

Example : $8 < m + 6 < 14$

*Rewrite the compound inequality using the word "**and**", then solve each inequality.*

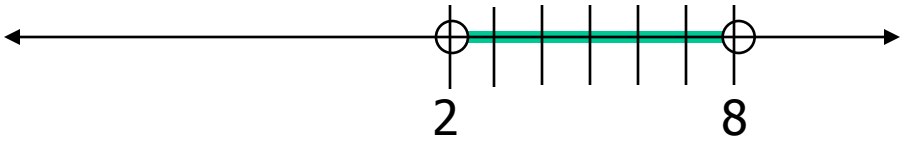
$8 < m + 6$ *and* $m + 6 < 14$

$2 < m$ $m < 8$

$m > 2$ *and* $m < 8$

$2 < m < 8$

Graph the solution:



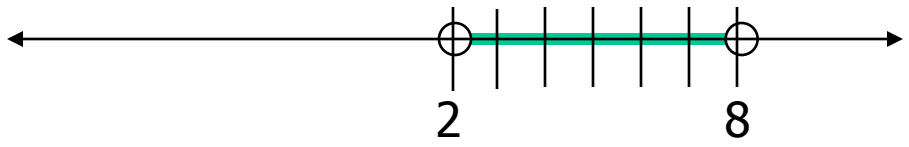
Method Two

Example: $8 < m + 6 < 14$

To solve the inequality, isolate the variable by subtracting 6 from all 3 parts.

$$8 < m + 6 < 14$$
$$-6 \quad -6 \quad -6$$
$$2 < m < 8$$

Graph the solution.



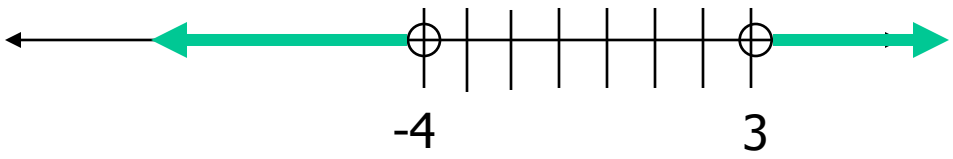
'or' Statements

Example: $x - 1 > 2$ or $x + 3 < -1$

$x > 3$ $x < -4$

$x < -4$ or $x > 3$

Graph the solution.



Solving an Absolute Value Inequality

- Step 1: Rewrite the inequality as a conjunction or a disjunction.
  - If you have a $<$ or $\leq$ you are working with a conjunction or an '***and***' statement.
    Remember: "*Less th**and***"
  - If you have a $>$ or $\geq$ you are working with a disjunction or an '***or***' statement.
    Remember: "*Great**or***"
- Step 2: In the second equation you must ***negate*** the right hand side and ***reverse*** the direction of the inequality sign.
- Solve as a compound inequality.

Example 1:

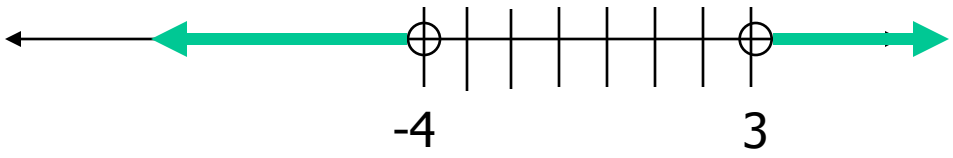
- $|2x + 1| > 7$
- $2x + 1 > 7$ or $2x + 1 > 7$
- $2x + 1 > 7$ or $2x + 1 < -7$
- $x > 3$ or $x < -4$

This is an '***or***' statement. (Great*or*). Rewrite.

In the 2nd inequality, *reverse* the inequality sign and *negate* the right side value.

Solve each inequality.

Graph the solution.



Example 2:

- $|x - 5| < 3$

This is an '***and***' statement. (Less th**and**).

- $x - 5 < 3$ and $x - 5 < 3$
- $x - 5 < 3$ and $x - 5 > -3$

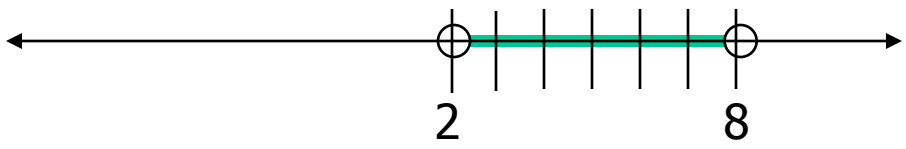
Rewrite.

In the 2nd inequality, *reverse* the inequality sign and *negate* the right side value.

- $x < 8$ and $x > 2$
- $2 < x < 8$

Solve each inequality.

Graph the solution.



Solve and Graph

- 1) $4m - 5 > 7$ or $4m - 5 < -9$
- 2) $3 < x - 2 < 7$
- 3) $|y - 3| > 1$
- 4) $|p + 2| < 6$